

**KAVAYITRI BAHINABAI CHAUDHARI NORTH
MAHARASHTRA UNIVERSITY, JALGAON**



**STRUCTURE AND SYLLABUS OF
S.Y. B. Sc. (MATHEMATICS)**

UNDER CHOICE BASED CREDIT SYSTEM (CBCS)

Effective from June 2019

**KAVAYITRI BAHINABAI CHAUDHARI
NORTH MAHARASHTRA UNIVERSITY, JALGAON**

**Syllabus for S. Y. B. Sc. (Mathematics)
Under Choice Based Credit System (CBCS)
Effective from June 2019**

The examination pattern is semester system for both the theory and practical papers. Each theory paper is of 100 marks (60 marks for external examination and 40 marks for internal examination) and practical paper is of 100 marks (60 marks for external examination and 40 marks for internal examination). The examination will be conducted at the end of each semester. Period of teaching for each theory paper is 30 clock hours and for practical paper is 60 clock hours.

COURSE STRUCTURE

Sem.	Course	Paper	Course Code with Title	Credits	No. Periods in Hour /week
III	MTHCC- C	Paper - 1	MTH 301: Calculus of Several Variables	2	2
		Paper - 2	MTH 302(A): Group Theory Or MTH -302(B): Theory of Groups and Codes	2	2
		Paper - 3	MTH 303: Practical paper based on MTH 301 and MTH 302	2	4
	SEC-1	SEC-1	MTH 304: Set Theory and Logic	2	2
IV	MTHCC- D	Paper - 1	MTH 401: Complex Variables	2	2
		Paper - 2	MTH 402(A): Differential Equations Or MTH-402 (B): Differential Equations and Numerical Methods	2	2
		Paper - 3	MTH 403: Practical paper based on MTH 401 and MTH 402	2	4
	SEC-2	SEC-2	MTH 404: Vector Calculus	2	2

Syllabus for S.Y. B.Sc. (Mathematics)

SEMESTER – III

MTH -301: Calculus of Several Variables (Period: 30 Clock hours)

Course Description:

This course provides an elementary level knowledge of functions of several variables, their limit continuity, Taylors expansion, differentiation and integration of functions of two or more variables.

Prerequisite Course(s): Preliminary knowledge of real analysis, functions of one variables and calculus.

General Objective:

This is the second course in the calculus series after a course of Calculus in F. Y. B. Sc. for science students. In this course we discuss functions of two and more variables along with their series expansions and extreme values. We also discuss integration techniques as well as applications of integrals.

Learning Outcomes:

Upon successful completion of this course the student will be able to understand:

- a) limit and continuity of functions of several variables
- b) fundamental concepts of multivariable Calculus.
- c) series expansion of functions.
- d) extreme points of function and their maximum, minimum values at those points.
- e) meaning of definite integral as limit as sums.
- f) how to solve double and triple integration and use them to find area by double integration and volume by triple integration.

Unit- 1: Functions of Two and Three Variables

Marks-15

- 1.1 Explicit and Implicit Functions
- 1.2 Continuity
- 1.3 Partial Derivatives
- 1.4 Differentiability
- 1.5 Necessary and Sufficient Conditions for Differentiability
- 1.6 Partial Derivatives of Higher Order
- 1.7 Schwarz's Theorem
- 1.8 Young's Theorem.

Unit-2: Jacobian, Composite Functions and Mean Value Theorems

Marks-15

- 2.1 Jacobian (Only for Two and Three Variable)
- 2.2 Composite Functions (Chain Rule)

- 2.3 Homogeneous Functions.
- 2.4 Euler's Theorem on Homogeneous Functions.
- 2.5 Mean Value Theorem for Function of Two Variables.

Unit -3: Taylor's Theorem and Extreme Values

Marks-15

- 3.1 Taylor's Theorem for Function of Two Variables.
- 3.2 Maclaurin's Theorem for Function of Two Variables.
- 3.3 Absolute and Relative Maxima & Minima.
- 3.4 Necessary Condition for Extrema.
- 3.5 Critical Point, Saddle Point.
- 3.6 Sufficient Condition for Extrema.

Unit -4: Double and Triple Integrals

Marks-15

- 4.1 Double Integrals by Using Cartesian and Polar Coordinates.
- 4.2 Change of Order of Integration.
- 4.3 Area by Double Integral.
- 4.4 Evaluation of Triple Integral as Repeated Integral.
- 4.5 Volume by Triple Integral.

Recommended Book:

Mathematical Analysis: S.C. Malik and Savita Arora. Wiley Eastern Ltd, New Delhi. 1992
(Chapter 15: Functions of several variables 1, 1.1, 1.2, 1.3, 1.4, 1.6, 2, 3, 3.1, 3.2, 4, 4.1, 5, 5.2, 6, 7.2, 9, 9.1, 10, 10.1, 10.2)

Reference Books –

- 1. Calculus of Several Variables by Schaum's Outline Series.
- 2. Mathematical Analysis by T. M. Apostol, Narosa Publishing House, New Delhi, 1985

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MTH -302(A): Group Theory (Period: 30 Clock hours)

Course Description:

This course provides an elementary level knowledge of algebraic structure like groups and rings.

Prerequisite Course(s): Preliminary knowledge of sets, functions and binary operations and number systems like Set of integers, rationals, reals and complex.

General Objective:

A primary objective of this course is to understand algebraic structures and their properties. Doing this one can use these structures to solve problems arises in many branches of Mathematics such as theory of equations, theory of numbers, Geometry etc. This enable students to grow their mathematical skill and used them to apply in many

branches of science. So, the main objective is to develop and maintain problem-solving skills of the students.

Learning Outcomes:

Upon successful completion of this course the student will be able to:

- a) understand group and their types which is one of the building blocks of pure and applied mathematics.
- b) understand Lagrange, Euler and Fermat theorem
- c) understand concept of automorphism of groups
- d) understand concepts of homomorphism and isomorphism
- e) understand basic properties of rings and their types such as integral domain and field.

Unit-1: Groups

Marks-15

- 1.1 Definition and Examples of a group.
- 1.2 Simple Properties of Group.
- 1.3 Abelian Group.
- 1.4 Finite and Infinite Groups.
- 1.5 Order of a Group.
- 1.6 Order of an Element and Its Properties.

Unit-2: Subgroups

Marks-15

- 2.1 Definition and Examples of Subgroups.
- 2.2 Simple Properties of Subgroup.
- 2.3 Criteria for a Subset to be a Subgroup.
- 2.4 Cyclic Groups
- 2.5 Normal subgroups and Coset Decomposition.
- 2.6 Lagrange's Theorem for Finite Group.
- 2.7 Euler's Theorem and Fermat's Theorem.

Unit-3: Homomorphism and Isomorphism of Groups

Marks-15

- 4.1 Definition and Examples of Group Homomorphism.
- 4.2 Properties of Group Homomorphism.
- 4.3 Kernel of a Group Homomorphism and its Properties.
- 4.4 Definition and Examples of Isomorphism.
- 4.5 Definition and Examples of Automorphism of Groups.
- 4.6 Properties of Isomorphism of Groups.

Unit -4: Rings

Marks-15

- 4.1 Definition and Simple Properties of a Ring.
- 4.2 Commutative Ring, Ring with unity, Boolean Ring.
- 4.3 Ring with zero divisors and without zero Divisors.
- 4.4 Integral Domain, Division Ring and Field. Simple Properties.

Recommended Book: -

1. University Algebra: N. S. Gopalakrishnan, New age international publishers, 2018. (Chapter 1: 1.3, 1.4, 1.5, 1.6,1.7, 1.8, 1.9)

Reference Books: -

1. Topics in Algebra: I. N. Herstein (John Wiley and Sons).
2. A first Course in Abstract Algebra: J. B. Fraleigh (Pearson).
3. A course in Abstract Algebra: Vijay K. Khanna and S. K. Bhambri, Vikas Publishing House Pvt. Ltd., Noida.

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MTH -302(B): Theory of Groups and Codes (Period: 30 Clock hours)

Course Description:

This course provides an elementary level knowledge of algebraic structure like groups and codes.

Prerequisite Course(s): Preliminary knowledge of Sets, functions and binary operations and number systems like Set of integers, rationals, reals and complex.

General Objective:

A primary need for the establishment of this course is to understand algebraic structures and their properties. Upon studying this one can use these sutures to solve problems arises in many branches of Mathematics and computer science such as theory of equations, theory of numbers, Geometry, theory of computations, cryptography etc. This enable students to grow their mathematical skill and used them to apply in many other branches of science and technology.

Learning Outcomes:

Upon successful completion of this course the student will be able to:

- a) understand group structures which is useful to understanding ideas of modern mathematics.
- b) understand solutions to polynomial equations
- c) understand permutation groups
- d) understand concepts of homomorphisms and isomorphisms
- e) Students will understand basic concepts in coding theory.

Unit-1: Groups

Marks-15

- 1.1 Definition and Examples of a group.
- 1.2 Simple Properties of Group.
- 1.3 Abelian Group.
- 1.4 Finite and Infinite Groups.
- 1.5 Order of a Group.
- 1.6 Order of an Element and Its Properties.

Unit-2: Subgroups

Marks-15

- 2.1 Definition and Examples of Subgroups.
- 2.2 Simple Properties of Subgroup.
- 2.3 Criteria for a Subset to be a Subgroup.

- 2.4 Cyclic Groups
- 2.5 Normal subgroups and Coset Decomposition.
- 2.6 Lagrange's Theorem for Finite Group.
- 2.7 Euler's Theorem and Fermat's Theorem.

Unit-3: Homomorphism and Isomorphism of Groups

Marks-15

- 3.1 Definition and Examples of Group Homomorphism.
- 3.2 Properties of Group Homomorphism.
- 3.3 Kernel of a Group Homomorphism and its Properties.
- 3.4 Definition and Examples of Isomorphism.
- 3.5 Definition and Examples of Automorphism of Groups.
- 3.6 Properties of Isomorphism of Groups.

Unit -4: Group Codes

Marks 15

- 4.1 Message, Word, (m, n) - Encoding Function, Code Words.
- 4.2 Detection of k or fewer errors, Weight, Parity Check Code
- 4.3 Hamming Distance, Properties of the Distance Function, Minimum Distance of an encoding function.
- 4.4 Group Codes.
- 4.5 (n, m) - Decoding function, Maximum Likelihood Decoding Function.
- 4.6 Decoding procedure for a Group Code given by a Parity Check Matrix.

Recommended Book: -

1. University Algebra: N. S. Gopalakrishnan, New age international publishers, 2018. (Chapter 1: 1.3, 1.4, 1.5, 1.6,1.7, 1.8, 1.9)
2. Discrete Mathematical Structures: Bernard Kolman, Robert C. Busby and Ross (Prentice Hall of India New Delhi, Eastern Economy Edition).

Reference Books: -

1. Topics in Algebra: I. N. Herstein (John Wiley and Sons).
2. A first Course in Abstract Algebra: J. B. Fraleigh (Pearson).

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MTH-303: Practical Course Based on MTH-301 and MTH-302

(Period: 60 Clock hours)

Practical No	Title of the Practical
1	Functions of two and three Variables
2	Jacobian, Composite Functions and Mean Value Theorems
3	Taylor's Theorem and Extreme Values
4	Double and Triple Integrals
5	Groups
6	Subgroups
7	Homomorphism and Isomorphism of Groups
8(A)	Rings
8(B)	Group Codes

List of Practical Problems

Practical 1: Functions of Two and Three Variables

1. Evaluate the limit, if it exists, for the following function

$$f(x, y) = \begin{cases} \frac{x^2y}{x^4 + y^2} & , \text{if } x^4 + y^2 \neq 0 \\ 0 & , \text{if } x = y = 0. \end{cases}$$

2. Let $f(x, y) = x \sin \frac{1}{x} + y \sin \frac{1}{y}$, $xy \neq 0$. Show that $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$.

3. Let $f(x, y) = \frac{x^2y^2}{x^4 + y^4 - x^2y^2}$, $(x, y) \neq (0,0)$. Verify that both the repeated limits exist and are equal, but simultaneous limit does not exist.

4. Show that the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & , \text{if } (x, y) \neq (0,0) \\ 0 & , \text{if } (x, y) = (0,0) \end{cases}$$

is continuous at the origin.

5. Let $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & , \text{if } (x, y) \neq (0,0) \\ 0 & , \text{if } (x, y) = (0,0) \end{cases}$

Show that both the first order partial derivatives exist at $(0, 0)$, but the function is not continuous thereat.

6. Discuss the continuity and differentiability at the origin of the function

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & , \text{if } (x, y) \neq (0,0) \\ 0 & , \text{if } (x, y) = (0,0) \end{cases}$$

7. Let $f(x, y) = \begin{cases} x^2 \tan^{-1} \left(\frac{y}{x} \right) - y^2 \tan^{-1} \left(\frac{x}{y} \right) & , \text{if } (x, y) \neq (0,0) \\ 0 & , \text{if } (x, y) = (0,0) \end{cases}$

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

8. Show that for the function

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & , \text{if } (x, y) \neq (0,0) \\ 0 & , \text{if } (x, y) = (0,0) \end{cases}$$

$f_{xy}(0,0) = f_{yx}(0,0)$, even though the conditions of Schwarz's theorem and Young's theorem are not satisfied.

9. Using differentials find approximate value of $\sqrt{(1.02)^2 + (1.97)^3}$.

10. Using differentials find approximate value of $(3.9)^2(2.05) + (2.05)^3$.

Practical 2: Jacobian, Composite functions and Mean value theorem

1. If $u = \cos x$, $v = \sin x \cos y$, $w = \sin x \sin y \cos z$, then show that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = (-1)^3 \sin^3 x \sin^2 y \sin z$$

2. If $z = f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$, $x = u + v$, $y = u - v$, then show that $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = \frac{u-v}{u^2+v^2}$.
3. If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
4. If z is function of x and y and if $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$.
5. If $z = f(u, v)$, where $u = 2x - 3y$ and $v = x + 2y$, then prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 3 \frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}$.
6. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. Hence, deduce that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$.
7. If $u = \sin^{-1}\left(\frac{x^2+2xy}{\sqrt{x-y}}\right)^{\frac{1}{5}}$, then find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ and $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$.
8. If $u = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{x-y}\right)$, then find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$.
9. Let $f(x, y) = x^2 y + 2xy^2$. Find the quadratic equation in θ by applying the mean value theorem applied to the line segment joining the points (1,2) to (3,3).
10. Let $f(x, y) = x^3 - xy^2$. Show that θ used in the mean value theorem applied to the points (2,1) and (4,1) satisfies the quadratic equation $3\theta^2 + 6\theta - 4 = 0$.

Practical 3: Taylor's theorem and Extreme values

1. Use Taylor's theorem of suitable order to expand $\sin x \sin y$ in the form $xy - \frac{1}{6}\{(x^3 + 3xy^2) \cos \theta x \sin \theta y + (y^3 + 3x^2y) \sin \theta x \cos \theta y\}$, $0 < \theta < 1$.
2. Show that the expansion of $\sin(xy)$ in powers of $(x - 1)$ and $\left(y - \frac{\pi}{2}\right)$ upto and including second degree terms is

$$1 - \frac{1}{8}\pi^2(x - 1)^2 - \frac{1}{2}\pi(x - 1)\left(y - \frac{\pi}{2}\right) - \frac{1}{2}\left(y - \frac{\pi}{2}\right)^2.$$

3. Using Maclaurin's expansion, prove that $e^{ax} \cos by = 1 + ax + \frac{a^2x^2 - b^2y^2}{2!} + \frac{a^3x^3 - 3ab^2xy^2}{3!}$.
4. Expand $e^x \tan^{-1}y$ about (1, 1) up to the second degree in powers of $(x - 1)$ and $(y - 1)$.
5. Find maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.
6. Discuss the extreme values of the function $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$.
7. Investigate maximum and minimum values of $f(x, y) = (x + y - 1)(x^2 + y^2)$.
8. Find the extreme values of $f(x, y) = xy(a - x - y)$.
9. Find the least value of the function $f(x, y) = xy + \frac{50}{x} + \frac{50}{y}$.

Practical -4: Double and Triple Integrals

1. Using double integration, find the area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$.
2. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, using double integral.
3. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dx dy dz$.
4. Using triple integration, find the volume of the sphere of radius a .
5. Evaluate $\int_0^1 \int_0^2 \int_0^3 (x + y + z) dx dy dz$.
6. Change the order of integration in $\int_0^4 \int_0^{\sqrt{4x-x^2}} f(x, y) dx dy$.
7. Draw a sketch of the region of integration
 - i) $\int_0^4 \int_0^{\sqrt{25-x^2}} f(x, y) dx dy$.
 - ii) $\int_{-1}^3 \int_{x^2}^{x+3} f(x, y) dx dy$.
8. Evaluate $\iint y dx dy$ over the area bounded by $y = x^2$ and $x + y = 2$.

Practical - 5: Groups

1. Let \mathbb{Q}^+ denotes the set of all positive rational numbers and for any $a, b \in \mathbb{Q}^+$, define $a * b = \frac{ab}{3}$. Show that $(\mathbb{Q}^+, *)$ is an abelian group.
2. Let $G = \{(a, b): a, b \in \mathbb{R}, a \neq 0\}$. Show that (G, \odot) is a non-abelian group, where $(a, b) \odot (c, d) = (ac, ad + b)$.
3. Let G be a group and $a \in G, n \in \mathbb{N}$. Show that $a^n = e$ if and only if $o(a) | n$.
4. Show that a group G is abelian if and only if $(ab)^2 = a^2b^2 \forall a, b \in G$.

5. In the group (\mathbb{Z}_7, \times_7) , find (i) $(\bar{3})^2$ ii) $(\bar{4})^{-3}$ iii) $o(\bar{3})$ iv) $o(\bar{4})$
6. In the group $(\mathbb{Z}_{11}, \times_{11})$, find (i) $(\bar{4})^3$ ii) $(\bar{5})^2$ iii) $o(\bar{9})$ iv) $o(\bar{7})$
7. Show that $G = \mathbb{R} - \{1\}$ is an abelian group under the binary operation $a * b = a + b - ab, \forall a, b \in G$
8. Prove that $G = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix} : x \text{ is a non-zero real number} \right\}$ is a group under matrix multiplication.
9. If G is a group such that $a^2 = e, \forall a, b \in G$, then show that G is abelian.
10. If in a group $G, a^5 = e$ and $aba^{-1} = b^2, \forall a, b \in G$, then find order of an element b .

Practical - 6: Subgroups

1. If G is a group, then show that the center of $G, Z(G)$, is a subgroup of G , where $Z(G) = \{x \in G : xa = ax, \forall a \in G\}$.
2. Show that (\mathbb{Z}_7, \times_7) is a cyclic group. Find all its generators, all its proper subgroups and order of every element.
3. Let $G = \{1, -1, i, -i, j, -j, k, -k\}$ be a group under multiplication and $H = \{1, -1, i, -i\}$ be its subgroup. Find all the left and right cosets of H in G .
4. Let A and B be two subgroups of a finite group G whose orders are relatively prime. Show that $A \cap B = \{e\}$.
5. Show that every proper subgroup of a group of order 77 is cyclic.
6. Find the remainder obtained when 15^{27} is divided by 8.
7. Find the remainder obtained when 33^{19} is divided by 7.
8. Let G be a group of all non-zero complex numbers under multiplication. Show that $H = \{a + ib : a^2 + b^2 = 1\}$ is a subgroup of G .
9. If H is subgroup of a group G and if the normalizer of $H, N(H) = \{g \in G : gHg^{-1} = H\}$, then prove that (a) $N(H)$ is a subgroup of G and (b) H is a normal subgroup of $N(H)$.
10. If G is a group and H is a subgroup of index 2 in G , then prove that H is a normal subgroup of G .

Practical - 7: Homomorphism and Isomorphism of Groups

1. Let $G = \{A : A \text{ is } n \times n \text{ matrix over } \mathbb{R} \text{ and } |A| \neq 0\}$, the group of non-singular matrices of order n over \mathbb{R} under matrix multiplication and let $\mathbb{R}^* = \mathbb{R} - \{0\}$, be the group of nonzero real numbers under multiplication. Define $f : G \rightarrow \mathbb{R}^*$ by $f(A) = |A|$, for all $A \in G$. Show that f is an onto group homomorphism and find its kernel.
2. If $G_1 = \{1, -1, i, -i\}$ is a group under multiplication and $G_2 = \{2, 4, 6, 8\}$ is a group under multiplication modulo 10, then show that G_1 and G_2 are isomorphic.
3. Let G be a group and $a \in G$. Show that $f_a : G \rightarrow G$ defined by $f_a(x) = axa^{-1}$, for all $x \in G$ is an automorphism.

4. Let G be a group and $f : G \rightarrow G$ be a map defined by $f(x) = x^{-1}$, for all $x \in G$. Prove that
 - (a) If G is abelian, then f is an isomorphism.
 - (b) If f is a group homomorphism, then G is abelian.
5. Let $G = \{a, a^2, a^3, \dots, a^{11}, a^{12} = e\}$ be a cyclic group of order 12 generated by a . Show that $f : G \rightarrow G$ defined by $f(x) = x^4, \forall x \in G$ is a group homomorphism. Find the kernel of f .
6. Let f and g be group homomorphisms from the group G into G . Show that $H = \{x \in G : f(x) = g(x)\}$ is a subgroup of G .
7. Prove that the mapping $f : \mathbb{C} \rightarrow \mathbb{C}_0$ such that $f(z) = e^z$ is a homomorphism of the additive group of complex numbers onto the multiplicative group of non-zero complex numbers. What is the kernel of f ?
8. Let G be a group of all matrices of the type $\left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a, b \in \mathbb{C} \text{ and } a^2 + b^2 = 1 \right\}$ under matrix multiplication and G' be a group of non-zero complex numbers under multiplication. Show that $f : G \rightarrow G'$ defined by $f\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\right) = a + ib$, is an isomorphism.

Practical – 8(A): Rings

1. (a) Show that $\mathbb{Z}_7 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}\}$ forms a ring under addition and multiplication modulo 7.
 (b) In the ring $(\mathbb{Z}_{10}, +_{10}, \times_{10})$, find all divisors of zero.
2. Show that $\mathbb{Z}[i] = \{a + ib : a, b \in \mathbb{Z}\}$, the set of Gaussian integers, forms an integral domain under usual addition and multiplication of complex numbers.
3. Show that $R = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ is an integral domain under usual addition and multiplication.
4. In the ring $(\mathbb{Z}_7, +_7, \times_7)$, find (i) $-(\bar{4} \times_7 \bar{6})$; (ii) $\bar{3} \times_7 (\overline{-6})$; (iii) $(\overline{-5}) \times_7 (\overline{-5})$ (iv) Units in \mathbb{Z}_7 ; (v) additive inverse of $\bar{6}$; (vi) zero divisors. Is \mathbb{Z}_7 a field or an integral domain? Justify.
5. Let \mathbb{R} be the set of all real numbers. Show that $\mathbb{R} \times \mathbb{R}$ forms a field under addition and multiplication defined by $(a, b) + (c, d) = (a + c, b + d)$ & $(a, b) \cdot (c, d) = (ac - bd, ad + bc)$.
6. If p is a prime number, then show that \mathbb{Z}_p is an integral domain.
7. Which of the following rings are integral domains? (i) \mathbb{Z}_{187} ; (ii) \mathbb{Z}_{61} ; (iii) $\mathbb{Z}_{2 \times 2}$. (iv) $(\mathbb{Z}, +, \cdot)$.

Practical – 8(B): Group Codes

1. Consider the (3,8) encoding function $e : B^3 \rightarrow B^8$ defined by $e(000) = 00000000$, $e(001) = 10111000$, $e(010) = 00101101$, $e(011) = 10010101$, $e(100) = 10100100$, $e(101) = 10001001$, $e(110) = 00011100$, $e(111) = 00110001$.
 - (a) Find the minimum distance of e .
 - (b) How many errors will e detect?
2. Show that the (3,6) encoding function $e : B^3 \rightarrow B^6$ defined by $e(000) = 000000$, $e(001) = 001100$, $e(010) = 010011$, $e(011) = 011111$, $e(100) = 100101$, $e(101) = 101001$, $e(110) = 110110$, $e(111) = 111010$ is a group code. Also find the minimum distance of e .
3. Compute: (a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
4. Let $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the (2, 5) group code $eH : B^2 \rightarrow B^5$.
5. Consider the parity check matrix: $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Decode the following words relative to a maximum likelihood decoding function associated with eH :
 a) 10100 b) 01101 c) 11011
6. Consider the parity check matrix: $H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Determine the coset leaders for $N = eH(B^3)$. Also compute the Syndrome for each coset leader and decode the code 001110 relative to maximum likelihood decoding function.
7. Let the (9,3) decoding function $d : B^9 \rightarrow B^3$ be defined by $d(y) = Z_1Z_2Z_3$, where $Z_i = 1$, if $\{y_i, y_i + 3, y_i + 6\}$ has at least two 1's
 = 0, if $\{y_i, y_i + 3, y_i + 6\}$ has less than two 1's, $i = 1, 2, 3$.
8. If $y \in B^9$, then determine $d(y)$, where (i) $y = 101111101$ (ii) $y = 100111100$.

Note: Practical problems based on each unit are not limited to the given ones, but any other related challenging and application-oriented problems may also be evaluated in the practical sessions.

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SEC -1

MTH 304: Set Theory and logic (Period: 30 Clock hours)

Course Description:

This course is an elementary skill development course for S. Y. B.Sc. students.

Prerequisite Course(s): Secondary school level knowledge of elementary mathematics.

General Objective: The general objectives are to acquire concepts of sets, relations, countable and uncountable sets; statements and truth values; concept of tautology, contradiction and quantifiers.

Learning Outcomes:

- a) Uses of the language of set theory, designing issues in different subjects of mathematics
- b) understand the issues associated with different types of finite and infinite sets via countable uncountable sets
- c) knowledge of the concepts and methods of mathematical logic, set theory, relation calculus, and concepts concerning functions which are included in the fundamentals of various disciplines of mathematics
- d) understanding the role of propositional and predicate calculus
- e) able to provide the logical mathematical reasoning, formulate theorems and definitions

Unit-1: Sets and Subsets

Marks-15

1.1 Finite Set and Infinite set

1.2 Equality of two Sets,

1.3 Null Set, Subset, Proper subset, Symmetric difference of two sets

1.4 Universal set, Power set, Disjoint sets,

1.5 Operation on sets: Union and Intersection

1.6 Venn diagram

1.7 Equivalent sets

1.8 Countable and uncountable sets

Unit-2: Relations and Functions

Marks-15

1.1 Product of sets

1.2 Relations, Types of relations, Reflexive, Symmetric, Transitive relations and Equivalence relations

1.3 Function, Types of functions, One-one, Onto, Even, Odd and Inverse function

1.4 Composite functions

Unit-3: Algebra of Propositions

Marks-15

2.1 Statements, Conjunction, Disjunction.

2.2 Negation, Conditional and Bi-Conditional statements, Propositions.

2.3 Truth table, Tautology and Contradiction.

2.4 Logical equivalence, Logical equivalent statements.

Unit-4: Quantifiers

Marks-15

3.1 Propositional functions and Truth sets.

3.2 Universal quantifier, Existential quantifier.

3.3 Negation of proposition which contain quantifiers, Counter examples.

Recommended book:

1. Set Theory and Related Topics by Schaum's outline Series (Chapter1, chapter 4, chapter 6: 6.2, 6.3, chapter 10)

Reference Books:

1. R.R.Halmons, Naïve Set Theory, Springer, 1974

2. E. Kamke, Theory of Sets, Dover Publishers,1950

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SEMESTER – IV

MTH -401: Complex Variables (Period: 30 Clock hours)

Course Description:

This course will improve basic and intermediate level knowledge of a special type of number system namely complex numbers and also discusses complex valued function with their integrations.

Prerequisite Course(s): Basic knowledge of Sets, functions, real valued functions, their limits and continuity and integrations.

General Objective:

A primary objective of this course is to make students aware of generalization of real number system and calculus. Analyticity and complex integrations are useful for applications. This course improves mathematical skill and ability to solve various integrations.

Learning Outcomes:

- a) The course is aimed to introduce the theory for functions of complex variables
- b) Students will understand the concept of analytic function
- c) Students will understand the Cauchy Riemann Equations
- d) Students will understand harmonic functions
- e) Students will understand complex integrations
- f) Students will understand calculus of residues.
- g) Students will acquire the skill of contour integrations.

Unit-1: Complex numbers

Marks-15

1.1 Complex numbers, modulus and amplitude, polar form

1.2 Triangle inequality and Argand's diagram

1.3 DeMoivre's theorem for rational indices and applications

1.4 n^{th} roots of a complex number

1.5 Elementary functions: Trigonometric functions, Hyperbolic functions of a complex variables (definitions only).

Unit-2: Functions of complex variables

Marks-15

2.1 Limits, Continuity and Derivative.

2.2 Analytic functions, A Necessary and sufficient conditions for analytic functions.

2.3 Cauchy Riemann equations.

2.4 Laplace equations and Harmonic functions

2.5 Construction of analytic functions

Unit-3: Complex integrations

Marks-15

3.1 Line integral and theorems on it.

- 3.2 Statement and verification of Cauchy-Goursat's Theorem.
- 3.3 Cauchy's integral formulae for $f(a)$, $f'(a)$ and $f^n(a)$
- 3.4 Taylor's and Laurent's series.

Unit-4: Calculus of Residues

Marks-15

- 4.1 Zeros and poles of a function.
- 4.2 Residue of a function
- 4.3 Cauchy's residue theorem
- 4.4 Evaluation of integrals by using Cauchy's residue theorem
- 4.5 Contour integrations of the type $\int_0^{2\pi} f(\cos \theta, \sin \theta) d\theta$ and $\int_{-\infty}^{\infty} f(x) dx$

Recommended book:

- 1. Complex Variables and Applications; J. W. Brown and R. V. Churchill. 7th Edition. (McGraw-Hill) (Chapter 1, chapter 2, chapter 3, chapter 4, chapter 6)

Reference Books:

- 1. Theory of Functions of Complex Variables: Shanti Narayan, S. Chand and Company New Delhi.
 - 2. Complex variables: Schaum's Outline Series.
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MTH-402(A): Differential Equations (Period: 30 Clock hours)

Course Description:

This course is of primary nature and here we introduce the students how the Differential Equations are formed and how to solve them using various methods.

Prerequisite Course(s): Basic knowledge of Real and complex valued functions, differentiations and integrations.

General Objective:

The main objective of this program is to cultivate mathematical aptitude among the students and nurture their interest towards problem solving aptitude by introducing various methods of solution of differential equations.

Learning Outcomes:

- a) Students will aware of formation of differential equations and their solutions
- b) Students will understand the concept of Lipschitz condition
- c) Students will understand method of variation of parameters for second order L.D.E.
- d) Students will understand simultaneous linear differential equations and method of their solutions
- e) Students will understand Pfaffian differential equations and method of their solutions
- f) Students will understand difference equations and their solutions

Unit-1: Theory of ordinary differential equations**Marks-15**

- 1.1 Lipschitz condition
- 1.2 Existence and uniqueness theorem
- 1.3 Linearly dependent and independent solutions
- 1.4 Wronskian definition
- 1.5 Linear combination of solutions
- 1.6 Theorems on i) Linear combination of solutions ii) Linearly independent solutions
iii) Wronskian is zero iv) Wronskian is non-zero
- 1.7 Method of variation of parameters for second order L.D.E.

Unit-2: Simultaneous Differential Equations**Marks-15**

- 2.1 Simultaneous linear differential equations of first order
- 2.2 Simultaneous D.E. of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
- 2.3 Rule I: Method of combinations
- 2.4 Rule II: Method of multipliers
- 2.5 Rule III: Properties of ratios
- 2.6 Rule IV: Miscellaneous

Unit-3: Total Differential or Pfaffian Differential Equations**Marks-15**

- 3.1 Pfaffian differential equations
- 3.2 Necessary and sufficient conditions for the integrability
- 3.3 Conditions for exactness
- 3.4 Method of solution by inspection
- 3.5 Solution of homogenous equation

Unit-4: Difference Equations**Marks-15**

- 4.1 Introduction, Order of difference equation, degree of difference equations
- 4.2 Solution to difference equation and formation of difference equations
- 4.3 Linear difference equations, Linear homogeneous difference equations with constant coefficients
- 4.4 Non-homogenous linear difference equation with constant coefficients

Recommended books:

1. Ordinary and Partial Differential Equation by M. D. Rai Singhania, S. Chand & Co. 18th Edition. (Chapter 1 and Chapter 2)
2. Numerical Methods by V. N. VEDAMURTHY and N. Ch. S. N. IYENGAR, Vikas Publishing House, New Delhi. (Chapter 10).

Reference Book:

1. Introductory course in Differential Equations by D. A. Murray, Longmans Green and co. London and Mumbai, 5th Edition 1997.

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MTH-402 (B): Differential Equations and Numerical Methods (Period: 30 Clock hours)

Unit-1 : Theory of ordinary differential equations

Marks-15

- 1.1 Lipschitz condition
- 1.2 Existence and uniqueness theorem
- 1.3 Linearly dependent and independent solutions
- 1.4 Wronskian definition
- 1.5 Linear combination of solutions
- 1.6 Theorems on i) Linear combination of solutions ii) Linearly independent solutions
iii) Wronskian is zero iv) Wronskian is non-zero
- 1.7 Method of variation of parameters for second order L.D.E.

Unit-2 : Simultaneous Differential Equations

Marks-15

- 2.1 Simultaneous linear differential equations of first order
- 2.2 Simultaneous D.E. of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
- 2.3 Rule I: Method of combinations
- 2.4 Rule II: Method of multipliers
- 2.5 Rule III: Properties of ratios
- 2.6 Rule IV: Miscellaneous

Unit-3 : Total Differential or Pfaffian Differential Equations

Marks-15

- 3.1 Pfaffian differential equations
- 3.2 Necessary and sufficient condition for integrability
- 3.3 Conditions for exactness
- 3.4 Method of solution by inspection
- 3.5 Solution of homogenous equation

Unit-4 : Numerical differentiation

Marks-15

- 4.1 Numerical Differentiation
- 4.2 Derivatives using Newtons forward interpolation formula
- 4.3 Derivatives using Newtons backward interpolation formula
- 4.4 Derivatives using Stirling's interpolation formula
- 4.5 Maxima and minima

Recommended books: -

1. Ordinary and Partial Differential Equation by M. D. Rai Singhania, S. Chand & Co. 18th Edition. (Chapter 1 and Chapter 2)
2. Numerical Methods by Dr. V. N. Vedamurthy and Dr. N. Ch. S. N. Iyengar, Vikas Publishing (Chapter 9)

Reference Books:

1. Introductory methods of Numerical Analysis, S.S. Sastry, Prentice Hall India, 12 th edition, New Delhi.
2. Differential equations, G.F. Simmons, Tata Mcgrawhill, 1972.

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MT-403: Practical course based on MTH-401, MTH-402
(Period: 60 Clock hours)

Practical No	Title of the Practical
1	Complex Numbers
2	Function of Complex Variable
3	Complex Integration
4	Calculus of Residues
5	Theory of ordinary differential equations
6	Simultaneous Differential Equations
7	Total (Pfaffian) Differential Equations
8(A)	Difference Equations
8(B)	Numerical Differentiation

List of Practicals

Practical-1: Complex Numbers

- Find the Modulus and principle value of the argument of $\frac{(1+i\sqrt{3})^{13}}{(\sqrt{3}-i)^{11}}$.
- If z_1, z_2, z_3 represents the vertices of an equilateral triangle, then prove that $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$.
- If $\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$, then show that
 - $\cos3\alpha + \cos3\beta + \cos3\gamma = 3\cos(\alpha + \beta + \gamma)$ and $\sin3\alpha + \sin3\beta + \sin3\gamma = 3\sin(\alpha + \beta + \gamma)$
 - $\cos2\alpha + \cos2\beta + \cos2\gamma = 0$ and $\sin2\alpha + \sin2\beta + \sin2\gamma = 0$
- Find all the values of $(1+i)^{\frac{1}{5}}$. Show that their continued product is $1+i$.
- Solve the equation $x^8 - x^4 + 1 = 0$.
- Determine the region in the Z-plane represented by $|z-3| + |z+3| = 10$.
- Using De Moivre's theorem express $\cos^6\theta$ in terms of cosines of multiple angles.
- If $|z_1| = |z_2| = |z_3| = 5$ and $\bar{z}_1 + \bar{z}_2 + \bar{z}_3 = 0$, then prove that $\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$.

Practical-2: Functions of Complex Variable

- Evaluate: $\lim_{z \rightarrow (1+i)} \frac{z^4+4}{z-1-i}$.
- If $f(z) = \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$, $z \neq i$ is continuous at $z = i$, then find the value of $f(i)$.
- Find an analytic function $f(z) = u + iv$ and express it in terms of z , if $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

4. Find an analytic function $f(z) = u + iv$ whose imaginary part is $v = e^x(x \sin y + y \cos y)$ using Milne Thomson Method.
5. Show that the real and imaginary parts of the function e^z satisfy C-R equations and they are harmonic.
6. Show that $u = \frac{1}{2} \log(x^2 + y^2)$ satisfies Laplace Equation. Find its harmonic conjugate.
7. If $f(z)$ is an analytic function with constant modulus, then show that $f(z)$ is a constant function.
8. Evaluate $\lim_{z \rightarrow e^{\frac{i\pi}{3}}} \frac{(z - e^{\frac{i\pi}{3}})z}{z^3 + 1}$.

Practical-3: Complex Integration

1. Evaluate $\int_C (y - x - 3x^2 i) dz$, where C is :
 - i. The straight-line joining $z = 0$ to $z = 1 + i$
 - ii. The straight-line joining $z = 0$ to $z = i$ first and then from $z = i$ to $z = 1 + i$.
2. Use the Cauchy Goursat theorem to obtain the value of $\int_C e^z dz$, where C is the circle $|z| = 1$ and hence deduce the following:
 - i. $\int_0^{2\pi} e^{\cos\theta} \sin(\theta + \sin\theta) d\theta = 0$
 - ii. $\int_0^{2\pi} e^{\cos\theta} \cos(\theta + \sin\theta) d\theta = 0$
3. Using Cauchy's Integral formula, evaluate $\int_C \frac{dz}{z^3(z+4)}$, where C is the circle $|z| = 2$.
4. Obtain the expansion of $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$, in the powers of z in the region
 - (i) $|z| < 2$
 - (ii) $2 < |z| < 3$
 - (iii) $|z| > 3$.
5. Prove that $\frac{1}{4z - z^2} = \sum_{n=0}^{\infty} \frac{z^{n-1}}{4^{n+1}}$, where $0 < |z| < 4$.
6. Verify Cauchy's integral theorem for $f(z) = z^2$ around the circle $|z| = 1$.
7. Evaluate $\int_{|z|=2} \frac{e^{2z}}{(z-1)^4} dz$ using Cauchy's integral formula.
8. Find the expansion of $f(z) = \frac{1}{(z^2+1)(z^2+2)}$ in powers of z , when $|z| < 1$.

Practical-4: Calculus of Residues

1. Find the residue of $f(z) = \frac{z^2 + 2z}{(z+1)^2(z+4)}$ at its poles.
2. Evaluate $\int_{|z|=3} \frac{e^z}{z(z-1)^2} dz$ by Cauchy's residue theorem.

3. Evaluate $\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$ by Cauchy's residue theorem, where C is
 (i) The circle $|z - 2| = 2$ (ii) The circle $|z| = 4$
4. Use the Contour integration to evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$.
5. Evaluate by Contour integration $\int_{-\infty}^{\infty} \frac{1}{x^4+13x^2+36} dx$.
6. Find the sum of residues of $f(z) = \frac{e^z}{z^2+a^2}$ at its poles.
7. Evaluate $\int_{|z|=2} \frac{dz}{z^3(z+4)}$ by Cauchy's residue theorem.
8. Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ by contour integration.

Practical 5: Theory of ordinary differential equations

1. Show that $f(x, y) = xy^2$ satisfies the Lipschitz condition on the rectangle $R: |x| \leq 1, |y| \leq 1$, but does not satisfy the Lipschitz condition on the strip $S: |x| \leq 1, |y| \leq \infty$.
2. Prove that $\sin 2x$ and $\cos 2x$ are solutions of $y'' + 4y = 0$ and these solutions are linearly independent.
3. Prove that $1, x, x^2$ are linearly independent. Hence, form the differential equation whose solutions are $1, x$ and x^2 .
4. Examine whether the set of functions $1, x^2, x^3$ are linearly independent or not.
5. Solve by method of variation of parameters $y'' + a^2y = \operatorname{cosec}(ax)$
6. Solve by method of variation of parameters $y'' + y - x = 0$
7. Show that functions $1 + x, x^2, 1 + 2x$ are linearly independent.
8. Examine whether e^{2x} and e^{3x} are linearly independent solution of differential equation $y'' - 5y' + 6y = 0$ or not?
9. Solve by method of variation of parameters, $y'' + 3y = \sec 3x$.

Practical 6 –Simultaneous Differential Equations

1. Solve : (i) $\frac{dx}{x^2z} = \frac{dy}{0} = \frac{dz}{-x^2}$ and (ii) $\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$
2. Solve : (i) $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{xyz-zx^2}$ and (ii) $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2-y^2)}$
3. Solve : $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$
4. Solve : $\frac{adx}{yz(b-c)} = \frac{bdy}{zx(c-a)} = \frac{cdz}{xy(a-b)}$
5. Solve : $\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$
6. Solve : $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$

7. Solve : $\frac{dx}{\sin(x+y)} = \frac{dy}{\cos(x+y)} = \frac{dz}{z}$
8. Solve : $\frac{dx}{z^2} = \frac{ydy}{xz^2} = \frac{dz}{xy}$
9. Solve : $\frac{dx}{y^2(x-y)} = \frac{dy}{-x^2(x-y)} = \frac{dz}{z(x^2+y^2)}$.

Practical - 7 : Total (Pfaffian) Differential Equations

1. Show that the following differential equations are integrable. Hence solve them
 - (i) $(y^2 + z^2 + x^2)dx - 2xydy - 2xzdz = 0$
 - (ii) $2yzdx + zxdy - xy(1 + z)dz = 0$
2. Solve : $yz^2(x^2 - yz)dx + zx^2(y^2 - xz)dy + xy^2(z^2 - xy)dz = 0$
3. Solve : $\frac{yz}{x^2+y^2}dx - \frac{xz}{x^2+y^2}dy - \tan^{-1}\frac{y}{x}dz = 0$
4. Solve : $zydx = zxdy + y^2dz = 0$
5. Solve : $(x^2 - yz)dx + (y^2 - xz)dy + (z^2 - xy)dz = 0$
6. Solve : $(2x^2 + 2xy + xz^2 + 1)dx + dy + 2zdz = 0$
7. Solve : $(y + z)dx + dy + dz = 0$
8. Show that the equation $yz^2(x^2 - yz)dx + zx^2(y^2 - xz)dy + xy^2(z^2 - xy)dz = 0$ is integrable. Is it exact? Verify.

Practical 8(A) : Difference Equations

1. Form the difference equation corresponding to the following general solution:
 - (a) $y = c_1x^2 + c_2x + c_3$
 - (b) $y = (c_1 + c_2n)(-2)^n$
2. Show that $y_x = c_1 + c_2 \cdot 2^x - x$ is a solution of difference equation $y_{x+2} - 3y_{x+1} + 2y_x = 1$.
3. Formulate the Fibonacci difference equation and solve it.
4. Solve the following difference equations:
 - (a) $y_{x+1} - 3y_x = 1$
 - (b) $y_{x+1} - 3y_x = 0, y_0 = 2$
5. Solve the following non-homogeneous linear difference equations:
 - (i) $y_{x+2} - 4y_x = 9x^2$
 - (ii) $\Delta y_x + \Delta^2 y_x = \sin x$
6. Solve: $y_{x+2} - 4y_{x+1} + 3y_x = 3^x + 1$.
7. Solve: $y_{x+2} - 4y_{x+1} + 4y_x = 3x + 2^x$.
8. Solve: $u_{x+2} - 5u_{x+1} + 6u_x = 36$.

Practical – 8 (B): Numerical differentiation

1. Find the first and second derivatives of the function tabulated below at $x = 1.9$

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.128	.544	1.296	2.432	4.0

2. Find first and second derivatives at $x = 0$ from the following table:

x	0	1	2	3	4	5
$f(x)$	4	8	15	7	6	2

3. Find the value of $\sec(310)$ from the following table:

x	31	32	33	34
$\sec(x)$	0.6008	0.6249	0.6494	0.6745

4. Find first derivative using Stirling's formula at $x = 0.5$:

x	0.35	0.4	0.45	0.5	0.55	0.6	0.65
$f(x)$	1.521	1.506	1.488	1.467	1.444	1.418	1.389

5. Find the maximum value of $f(x)$ from the following table:

x	3	4	5	6	7	8
$f(x)$	0.205	0.240	0.259	0.262	0.250	0.224

6. Find the maximum value of y from the following table:

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

7. Find the first derivative at $x = 4$ by using Stirling's formula:

x	1	2	3	4	5	6
$f(x)$	1	3	7	13	21	31

8. Find the maximum and minimum values of $f(x)$:

x	0	1	2	3	4	5
$f(x)$	0	0.25	0	2.25	16.0	56.25

Note: Practical problems based on each unit are not limited to the given ones, but any other related challenging and application-oriented problems may also be evaluated in the practical sessions.

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SEC- 2

MTH 404: Vector Calculus (Period: 30 Clock hours)

Course Description:

This is a skill development course of vector algebra and its calculus for S. Y. B.Sc. students

Prerequisite Course(s): Secondary school level knowledge of elementary physics and mathematics.

General Objective: The general objectives are to acquire skills of vectors algebra, vector valued functions, operators like del and curl and line and surface integrals.

Learning Outcomes:

- a) understand scalar and vector products
- b) understand vector valued functions and their limits and continuity and use them to estimate velocity and acceleration of partials.
- c) Calculate the curl and divergence of a vector field.
- d) Set up and evaluate line integrals of functions along curves.

Unit -1: Product of Vectors

Marks-15

- 1.1 Scalar Product
- 1.2 Vector Product
- 1.3 Scalar Triple Product
- 1.4 Vector Product of Three Vectors
- 1.5 Reciprocal Vector

Unit-2: Vector functions

Marks-15

- 1.1 Vector functions of a single variable.
- 1.2 Limits and continuity.
- 1.3 Differentiability, Algebra of differentiation.
- 1.4 Curves in space, Velocity and acceleration.
- 1.5 Vector function of two or three variables.
- 1.6 Limits, Continuity, Partial Differentiation

Unit-3: The Vector Operator Del

Marks-15

- 2.1 The vector differentiation operator del.
- 2.2 Gradient.
- 2.3 Divergence and curl.
- 2.4 Formulae involving del. Invariance.

Unit-4: Vector Integration

Marks-15

- 3.1 Ordinary integrals of vectors.
- 3.2 Line integrals.
- 3.3 Surface integrals.

Recommended Book:

1. Vector Analysis by Murray R Spiegel, Schaum's Series, McGraw Hill Book Company.

Reference Book:

1. Vector Calculus by Shanti Narayan and P.K. Mittal, S. Chand & Co., New Delhi

**KAVAYITRI BAHINABAI CHAUDHARI
NORTH MAHARASHTRA UNIVERSITY, JALGAON**

Equivalence courses for S. Y. B. Sc. (Mathematics)

Effective from 2019

Semester	Old course (June 2016)	New course (June 2019)
Sem-I	MTH 231 : Calculus of Several Variables	MTH 301 : Calculus of Several Variables
	MTH 232(A): Algebra	MTH 302(A) : Group Theory
	MTH 232(B): Theory of Groups	MTH -302(B): Theory of Groups and Codes
	MTH 233 : Practical Course based on MTH-232 & MTH-232	MTH 303 : Practical paper based on MTH 301 & MTH 302
Sem-II	MTH 241 : Complex Variables	MTH 401 : Complex Variables
	MTH 242(A): Differential Equations	MTH 402 : Differential Equations
	MTH 242(B): Differential and Difference Equations	MTH-402 (B): Differential Equations and Numerical Methods
	MTH 243 : Practical Course based on MTH-241 & MTH-242	MTH 403 : Practical paper based on MTH 401 & MTH 402